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13. ABSTRACT (Maximum 200 words)
Frequency hopping is the prevailing spread spectrum method in military communications, largely due to its low probability of detection and interception. In this project we developed a novel signal processing scheme for code-blind reception of multiple frequency hopped transmission over multipath channels. This techniques is based on the principle of dynamic programming and/or expectation-maximization, coupled with multidimensional harmonic and low rank analysis. It is able to jointly estimate hop timing, hop frequency, and directions of multiple frequency hopped signals in the presence of frequency collisions, without the knowledge of signal hop patterns. The method can also be used to obtain locate information and operation characteristic of active frequency hopped jammers. The associated identifiability of 2-D frequency estimation is also studied.

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FINAL REPORT

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A Project Abstract

Frequency hopping (FH) is the prevailing spread spectrum method in military communications, largely due to its low probability of detection and interception. In this project we developed a novel signal processing scheme for code-blind reception of multiple frequency hopped transmissions over multipath channels. This technique is based on the principle of dynamic programming and/or expectation-maximization, coupled with multidimensional harmonic and low-rank analysis. It is able to jointly estimate hop timing, hop frequency, and direction-of-arrival (DOA) of multiple FH signals in the presence of frequency collisions, without the knowledge of signal hop patterns. The method can also be used to obtain locate information and operation characteristics of active FH jammers. The associated identifiability of 2-D and multidimensional frequency estimation is also investigated.

B Technical Report

B.1 Problem Statement and Summary of Major Results

This project studied the problem of detection, localization, and tracking of multiple frequency hopped signals in multipath channels, without the knowledge of their hop codes and hop timing. Interception and localization of multiple FH signals are challenging problems that feed into multiple facets of military communications, from interception of noncooperative communications to jammer mitigation. On the technical side, the problems are challenging not only because hopping patterns and DOAs are unknown, but also other parameters such as frequency bin-width, hop-rate, and timing are at least partially unknown in a realistic scenario. Carrier hopping means that one has to deal with switching exponentials, rather than pure exponentials; and it also induces hopping in the receive antenna array spatial steering vectors, due to wavelength-dependent phase shifting from one array element to another.

Many signal processing techniques have been developed for blind interference suppression in frequency hopping using an antenna array, e.g., [26,29–31]. These approaches aim for interference suppression rather than joint multiuser detection and hopping pattern identification, they all require at least knowledge of the hopping pattern of a given signal of interest, and their interference nulling capability is bounded by the degrees of freedom in the adaptive array. On the other hand, several papers have been published on the subject of (joint) multiuser detection for frequency hopping systems, e.g., [4, 18]. These assume, among other things, that the hopping patterns of *all signals* are known to the receiver, hence clearly not applicable in a noncooperative scenario.

Without assuming the knowledge of hop patterns, several methods have been proposed for blind/semi-blind hop timing and frequency estimation. For example, assuming known hop rate, channelized receivers have been proposed for semi-blind hop timing estimation (knowledge of frequency channelization is required) for the *single* user case [2, 25], as well as the *multiuser* case [1]. However, the performance of those receivers degrades rapidly if the channelization is imperfect. [32] considers adaptive source localization and blind beamforming for FH signals without the knowledge of hopping patterns. The algorithm requires rough synchronization with the desired

signal's hop interval and is restricted to the use of very special array geometries, and the number of users that can be resolved is bounded by the number of sensor array elements. In our preliminary work [17], a first step is used to separate multiple FH signals and then a single user is tracked based on 1-D harmonic retrieval.

The major results of this project are summarized as follows.

1. We developed the DP-2DHR algorithm, based on the principle of dynamic programming (DP) coupled with 2-D harmonic retrieval (2-D HR), to jointly estimate the hop timing, hop frequency, DOA, of multiple FH signals in multipath channels with possible frequency collisions. The algorithm does not require the knowledge of hop patterns or channelization.

2. Using a multiple invariance sensor array, we developed the DP-TALS algorithm with low-rank decomposition of three-dimensional arrays, for joint timing, frequency, and 2-D DOA estimation of multiple FH signals without hop pattern knowledge.

3. Relying the principle of expectation-maximization (EM), we developed a low-complexity EM algorithm for joint hop timing and frequency estimation by exploiting the inherent data structure of multiple frequency hopped transmission.

4. We advanced the theory of identifiability of 2-D and multidimensional frequency estimation, and proved the most relaxed identifiability bound of N-D frequency estimation to date. An eigenvector-based algebraic algorithm for 2-D frequency estimation was also developed, which achieves the identifiability bound, and offers asymptotically optimal performance.

B.2 The DP-2DHR Algorithm for Timing and Frequency Estimation

B.2.1 Data Modeling

Suppose an M-element uniform linear array (ULA) receives FH transmission from d sources. Each far field FH signal is from a nominal DOA with negligible angel spread. The array steering vector in response to a signal from direction α is $\mathbf{a}(\theta) = [1, \theta, \ldots, \theta^{M-1}]^T$, where $\theta = e^{j2\pi\Delta\sin(\alpha)}$. The received signal is sampled at a sampling rate of $1/T$ (T is normalized to 1), and the $M \times 1$ signal vector collected at the ULA output at sampling time n can be expressed as

$$\boldsymbol{x}(n) = \sum_{r=1}^{d} \boldsymbol{a}(\theta_r^{(p)})\beta_r^{(p)} s_r(n) + \boldsymbol{w}(n), \tag{1}$$

where $s_r(n) = e^{j\omega_r^{(p)} n}$, and $\omega_r^{(p)}$ is the frequency of the transmitted signal from the r-th user during its p-th hop. Note that the baseline separation Δ (measured in wavelength units) is frequency dependent, hence so are the steering vectors. For notational clarity, sometimes we do not explicitly denote this dependence as long as it is clear from the context. The transmitted signals can be fast or slow frequency hopping, with FSK or linear modulation. $\beta_r^{(p)}$ is the complex path loss for the r-th user during its p-th hop that collects the (frequency-dependent) channel attenuation; the signal's initial phase $\phi_r^{(p)}$ is also absorbed into $\beta_r^{(p)}$. Here the carrier shifts due to hopping or

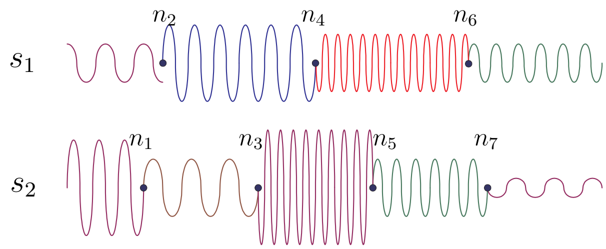

Figure 1: An example of two frequency-hopped signals.

symbol modulation are treated as conceptually equivalent, albeit of different magnitude. $\mathbf{w}(n)$ is complex white Gaussian noise with variance σ^2. Suppose N samples (snapshots) are collected at the array output, then the received data matrix can be written as $\boldsymbol{X} = [\boldsymbol{x}(0) \ \cdots \ \boldsymbol{x}(N-1)]$.

The objective here is to estimate hop timing (i.e., hop instants) and hop frequency sequences of all transmitted signals from $\boldsymbol{X}$, in the presence of possible collisions in some time segments, without the knowledge of signals' hop patterns or hop rates. In model (1) we assume a single-path transmitter-receiver propagation for each signal. Later we generalize it to incorporate multipath propagation with small delay spread. We also assume that the number of signals (and, in a multi-path scenario, the total number of paths for all signals) has already been estimated by an appropriate source enumeration method, such as rank criteria (e.g., SVD) or information theoretic criteria (e.g., AIC, MDL).

For simplicity of exposition, let us focus on an FH system where there are two active FH signals. As shown in Figure 1, $s_1(t)$ and $s_2(t)$ may have different hop rates and hop timing. Let n_i, $i = 0, \ldots, K-1$, be the *system-wide* hop instants ($n_0 = 0$ and $n_K = N$ by convention). We assume that within time period of interest, the total number of hops of all signals is bounded above by $K-1$ (such a bound could be deduced from the spectrogram of the data, and need not be tight).

Between any two *system-wide* consecutive hop instants, e.g., n_i and n_{i+1}, there are only two temporal frequencies involved. During such a time segment, the received data may be written as

$$\boldsymbol{X}_i = [\boldsymbol{x}(n_i) \ \cdots \ \boldsymbol{x}(n_{i+1}-1)] = \boldsymbol{A}_i \boldsymbol{B}_i \boldsymbol{S}_i^T + \boldsymbol{W}_i, \tag{2}$$

where $\boldsymbol{A}_i = [\boldsymbol{a}(\theta_1^{(p)}) \ \ \boldsymbol{a}(\theta_2^{(q)})]$, $\boldsymbol{B}_i = \mathrm{diag}(\beta_1^{(p)}, \beta_2^{(q)})$, and the subscript $_i$ is a time index indicating that the time segment is delimited between n_i and $n_{i+1}-1$, i.e., the i-th *system-wide* dwell. In (2), the signal matrix $\boldsymbol{S}_i$ is defined as

$$\boldsymbol{S}_i = \begin{bmatrix} e^{j\omega_1^{(p)} n_i} & e^{j\omega_1^{(p)}(n_i+1)} & \ldots & e^{j\omega_1^{(p)}(n_{i+1}-1)} \\ e^{j\omega_2^{(q)} n_i} & e^{j\omega_2^{(q)}(n_i+1)} & \ldots & e^{j\omega_2^{(q)}(n_{i+1}-1)} \end{bmatrix}^T,$$

and $\boldsymbol{W}_i$ is the corresponding noise matrix. Here we assume user 1 and user 2 are in their p-th and q-th hops respectively during this time segment, and $\boldsymbol{a}(\theta_1^{(p)})$ and $\mathbf{a}(\theta_2^{(q)})$ are the antenna steering vectors corresponding to $\omega_1^{(p)}$ and $\omega_2^{(q)}$. Since both $\mathbf{A}_i$ and $\boldsymbol{S}_i$ are Vandermonde matrices, the estimation of DOAs and frequencies from $\boldsymbol{X}_i$ in (2) is in fact a 2-D constant modulus harmonic retrieval problem, and there are two frequency components along each of the spatial and temporal dimensions. If d users are active in the system, a similar 2-D harmonic mixture model

can be obtained except that the number of frequency components in such a time segment along each dimension is d. Recently, improved identifiability results and algorithms regarding 2-D HR have been developed [6, 8, 11, 14, 15]. We use the MDF algorithm in [15] for the purpose of 2-D frequency estimation here.

B.2.2 Joint Timing and Frequency Estimation

The key idea behind our proposed method of joint hop timing and frequency estimation is that between any two hypothesized *system-wide* hops, the data follow a 2-D harmonic model. Hence for a hypothesized set of hops (that is, including all hops of all users in the system), 2-D harmonic retrieval methods can be used to estimate model parameters, and subsequently calculate model fit. If one operates under an upper bound on the total (system-wide) number of hops, then system stage can be defined as the number of allowable hops, and state can be defined as the hop instant, hence dynamic programming can be used to find the optimal hop sequence and associated model parameters per dwell [16]. With d users and a budget of $K-1$ hops, define

$$\begin{aligned}
\mathbf{n} &= [n_1, \ldots, n_{K-1}],\\
\boldsymbol{\alpha} &= [\alpha_1^{(0)}, \ldots, \alpha_1^{(K-1)}, \ldots, \alpha_d^{(0)}, \ldots, \alpha_d^{(K-1)}],\\
\boldsymbol{\beta} &= [\beta_1^{(0)}, \ldots, \beta_1^{(K-1)}, \ldots, \beta_d^{(0)}, \ldots, \beta_d^{(K-1)}],\\
\boldsymbol{\omega} &= [\omega_1^{(0)}, \ldots, \omega_1^{(K-1)}, \ldots, \omega_d^{(0)}, \ldots, \omega_d^{(K-1)}],
\end{aligned}$$

as the vectors of hop timing, DOAs, complex frequency-dependent attenuations, and hop frequencies. Joint maximum likelihood estimation of $\boldsymbol{n}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\omega}$ from $\boldsymbol{X}$ amounts to minimizing

$$J(\hat{\boldsymbol{n}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\omega}}) = \sum_{i=0}^{K-1} \|\mathbf{X}_i - \hat{\mathbf{X}}_i\|_F^2 \tag{3}$$

over $\hat{\boldsymbol{n}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\omega}}$, where $\hat{\boldsymbol{X}}_i$ is the reconstructed 2-D harmonic mixture based on ML parameter estimates (DOAs, complex amplitudes, and carrier frequencies), obtained in each time segment defined by hypothesized $\hat{n}_i$ and $\hat{n}_{i+1}$, assuming a 2-D harmonic mixture model for the received data during this segment. Since K will typically be higher than the true number of hops in the available samples, we include a "parking stage" in the DP program to account for the possibility of unused hops. In the presence of noise, however, DP will typically use any extra hops available to track minor noise-induced variations. Such variations can be relatively easily detected after DP, for frequencies before and after such hops will be approximately equal.

From the MDF estimates, we form

$$\hat{\boldsymbol{x}}(n) = \sum_{r=1}^{d} \boldsymbol{a}(\hat{\theta}_r)\hat{\beta}_r e^{j\hat{\omega}_r n},$$

for $n_i \leq n < n_{i+1}$; here, $\hat{\theta}_r = e^{j2\pi\Delta\sin(\hat{\alpha}_r)}$, and the matrix $\hat{\mathbf{X}}_i$ is constructed from $\hat{\mathrm{x}}(n)$ in the same form as $\mathbf{X}_i$ in (2). Define $\Lambda_i[n_i, n_{i+1}-1]$, for $0 \leq i \leq K-1$, as the cost function for the time segment $n_i \leq n < n_{i+1}$

$$\Lambda_i[n_i, n_{i+1}-1] = \|\boldsymbol{X}_i - \hat{\boldsymbol{X}}_i\|_F^2. \tag{4}$$

Table 1: The DP-2DHR Algorithm

1. **Initialization**
   Let $k = 1$, compute $\Gamma_k(L)$ for $L = 2, \ldots, N - 3K + 2$ using Eqns. (4) and (5).

2. **Recursion**
   For $2 \leq k \leq K - 1$, compute $\Gamma_k(L)$ with $L = 3k - 1, \ldots, N - 3K + 3k - 1$, using Eqn. (6) with $3k - 3 \leq n_{k-1} < L - 2$; For $k = K$, compute $\Gamma_k(L)$ with $L = N - 1$.

   For each L, denote the value of n_{k-1} that minimizes $\Gamma_k(L)$ as $n_{k-1}(L)$, and denote the corresponding $\hat{\boldsymbol{\alpha}}_{k-1}$, $\hat{\boldsymbol{\beta}}_{k-1}$, $\hat{\boldsymbol{\omega}}_{k-1}$ as $\hat{\boldsymbol{\alpha}}_{k-1}(L)$, $\hat{\boldsymbol{\beta}}_{k-1}(L)$, and $\hat{\boldsymbol{\omega}}_{k-1}(L)$, respectively.

3. **Backtracking**
   The maximum likelihood estimates of hop instants are obtained by using the backward recursion, i.e., $\hat{n}_i = n_i(\hat{n}_{i+1} - 1)$, for $i = K - 2, K - 3, \ldots, 1$, initialized by $\hat{n}_{K-1} = n_{K-1}(N - 1)$. Similarly, the corresponding DOA, amplitude, and frequency estimates of each segment can be obtained by their respective backward recursions.

Furthermore, to solve the minimization problem in (3) by DP, we define

$$\Gamma_k(L) = \min_{\substack{n_1, \ldots, n_{k-1} \\ n_0 = 0, n_k = L+1}} \sum_{i=0}^{k-1} \Lambda_i[n_i, n_{i+1} - 1], \tag{5}$$

where $0 < n_1 < \cdots < n_{k-1} < L$. Eqn. (5) can be viewed as the minimization problem of finding the best fit for a subset of the data of size $M \times (L+1)$ when a total number of $k-1$ hops is allowed. Hence $\Gamma_K(N-1)$ is the minimum of $J(\hat{\boldsymbol{n}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\phi}})$. From (5), a recursion for the minimum can be developed as

$$\Gamma_k(L) = \min_{n_{k-1}} \Big(\Gamma_{k-1}(n_{k-1} - 1) + \Lambda_{k-1}[n_{k-1}, L] \Big). \tag{6}$$

This means that for a data matrix of size $M \times (L+1)$, the minimum error for k segments (i.e., $k-1$ hop instants) is the minimum error for the first $k-1$ segments that end at $n = n_{k-1} - 1$, and the error contributed by the last segment from $n = n_{k-1}$ to $n = L$. The solution of the minimization of (3) is for $k = K$ and $L = N - 1$, which yields the joint estimates of hop timing, DOAs, and frequencies of all signals.

Assuming that the minimum length of a segment is three samples, the procedure to compute the solution by the DP and 2-D Harmonic Retrieval (DP-2DHR) algorithm is summarized in Table 1. Note that frequencies and complex amplitudes of different segments pertaining to a particular signal can be associated via their corresponding DOA parameters, since for a single segment,

frequency and DOA parameters pertaining to one signal are paired up automatically by the MDF algorithm. Depending on different transmission schemes, the application of the DP-2DHR method may slightly vary as described in the following cases.

1. Slow frequency hopping (SFH) with M-ary FSK modulation: Frequency changes due to baseband modulation are usually much smaller than those due to carrier hopping. Hence symbol rate and hop rate can be obtained from the result of DP, and consequently symbol recovery is possible.

2. SFH with M-ary PSK or M-ary QAM modulation: During one hop dwell, frequency is constant, but the complex amplitudes are different from symbol to symbol due to modulation (recall that for one hop dwell, the effect of channel on the complex amplitudes is constant). Hop timing can be detected from frequency change. Hence symbol rate and hop rate are distinguishable from the result of DP.

3. SFH with GMSK modulation: A GMSK signal is not a pure exponential in one symbol period. However, narrowband GMSK can be well-approximated by a pure exponential for our purpose.

4. Fast frequency hopping (FFH): The DP-2DHR method is applicable for hop timing and hop frequency sequence estimation. However, additional information is needed for symbol detection, e.g., symbol period and symbol synchronization are required since the DP-2DHR can only provide chip synchronization in this case.

Next we present the numerical simulation results to demonstrate the proposed DP-2DHR algorithm for joint hop timing and frequency estimation in the presence of frequency collisions. Two FH signals with DOAs $[12°,\ 17°]$ are simulated, each hopping with different hop timing. The receiver array has $M = 6$ antennas, with baseline separation of $\lambda/2$ at $f_c = 1$ GHz. With $M = 6$, the array has a 3dB beamwidth of about $28°$ so that the two sources, separated by $5°$, are not directly resolvable. A hopping frequency band of bandwidth 8 MHz is occupied by 32 frequency channels with 0.25 MHz channel spacing. The received signal is well-modeled as narrow-band. For simplicity of illustration, hop rate is set the same as symbol rate (125 Kbps). At the receiver, the complex antenna outputs are sampled at a rate of 8 MHz after down-conversion, and $N = 48$ complex samples are collected at each antenna, resulting in a 6 μs long analysis window, hence each signal hops at most once within this window. SNR is defined as

$$\text{SNR} := 10 \log_{10} \left(\frac{||\boldsymbol{X}||_F^2}{MN\sigma^2} \right), \tag{7}$$

where the noise variance $\sigma^2 = N_0 B$, and B is the processing signal bandwidth.

An example for the FH-FSK case is shown in Table 2 and Figure 2. In this example, two BFSK signals begin in different bins; then signal 1 hops to the same bin as signal 2; then signal 2 hops out of his original bin and into a new bin. This gives three segments: the first and the third without collisions, and the second with collisions. Table 2 gives the DOA estimation results for the three segments and Figure 2 gives the corresponding results of hop timing and frequency estimation

Table 2: DP-2DHR: DOA estimation of two FH-FSK signals.

	True DOA	Estimated DOA		
		1st Seg.	2nd Seg.	3rd Seg.
Signal 1	12°	12.24°	13.23°	12.69°
Signal 2	17°	17.35°	16.10°	17.16°

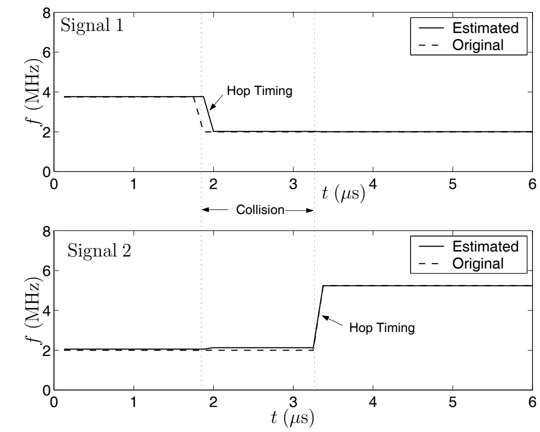

Figure 2: DP-2DHR: multiuser tracking of FH-FSK signals (SNR=10dB).

for the two signals. SNR is 10dB. We assume that any mobility-induced changes in DOA are negligible within the analysis window. Thus, varying hop frequencies are associated with different signals via their corresponding window invariant DOA parameters. The results show that DOA, hop timing and frequency estimates are close to the respective true values even in the presence of collisions. They also demonstrate that good estimates can be obtained, based on measurements of duration less than one symbol period; this implies that the algorithm is capable of blind multiuser tracking at moderate to heavy loads.

Figure 3 depicts the Root Mean Square Error (RMSE) of DP-2DHR hop timing and frequency estimates in the presence of collisions. The RMSE is obtained via Monte Carlo simulation. For each realization, each of the two FH-FSK signals hops once within the observation window. Hop timing is randomly generated, and frequencies are also randomly selected from the 32 candidate bins with the constraint that there is always one collision in the three hop-free segments. The RMSE vs. SNR performance shown in Figure 3 indicates that the DP-2DHR algorithm performs quite well even in the low SNR regime, given the fact that the signals are tracked in a situation where hop pattern, rate, and timing are all unknown.

B.2.3 Multipath Channels with Small Delay Spread

The DP-2DHR method developed in Section B.2.2 assumes single-path transmitter-receiver propagation for each frequency hopped signal. When the signal bandwidth is greater than the channel coherence bandwidth, channel effects due to multipath propagation cannot be ignored. Multipath reflections create fictitious sources in the spatial dimension, as well as unknown delay spread in

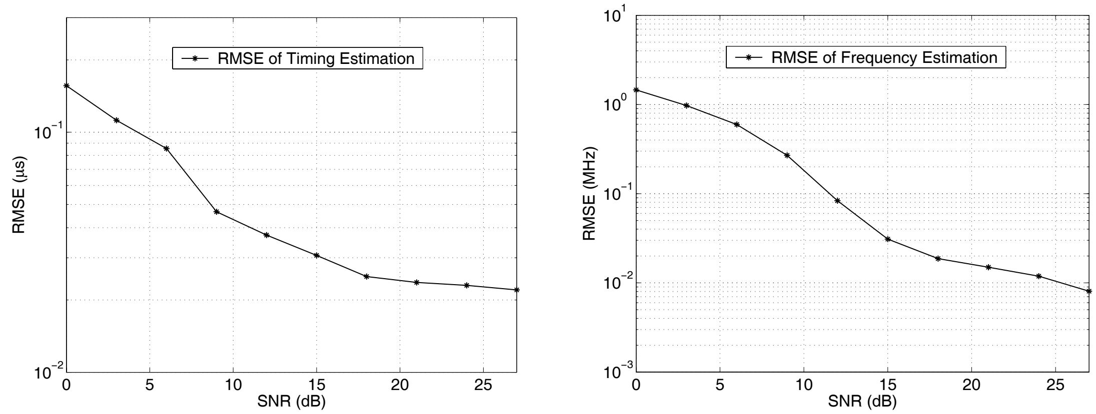

Figure 3: DP-2DHR: RMSE of timing and frequency estimation vs. SNR.

the temporal dimension. Few techniques can be found in the literature on blind parameter estimation for FH signals in multipath channels, e.g., [7], which assumes a fixed but unknown header in each packet of the FH signal. In this section we show that the DP-2DHR method can be extended to blindly estimate hop timing and frequency of multiple FH signals in multipath channels with unknown but small delay spread.

Suppose the r-th signal arrives at the ULA from L_r distinct paths due to multipath propagation, then (1) can be extended to

$$\boldsymbol{x}(n) = \sum_{r=1}^{d}\sum_{l=1}^{L_r} \boldsymbol{a}\left(\theta_{rl}^{(p)}\right)\beta_{rl}^{(p)} s_r\left(n-\tau_{rl}\right) + \boldsymbol{w}(n) \tag{8}$$

$$= \sum_{r=1}^{d}\sum_{l=1}^{L_r} \boldsymbol{a}\left(\theta_{rl}^{(p)}\right)\tilde{\beta}_{rl}^{(p)} e^{j\omega_r^{(p)} n} + \boldsymbol{w}(n) \tag{9}$$

for $n = 0, \ldots, N-1$, where $\tilde{\beta}_{rl}^{(p)} = \beta_{rl}^{(p)} e^{j\omega_r^{(p)}\tau_{rl}}$. Here we assume that the delay spread for a given signal is small so that time delay can be approximated by phase shift.

Between any two *system-wide* consecutive hop instants, e.g., n_i and n_{i+1}, the received data can be expressed in matrix form $\boldsymbol{X}_i = \boldsymbol{A}_i \boldsymbol{B}_i \boldsymbol{S}_i^T + \boldsymbol{W}_i$, which is a 2-D harmonic mixture model that is essentially the same as (2), with the difference that the number of frequency components in such a time segment along each dimension is $L_1 + L_2 + \cdots + L_d$. Some frequency components in the time dimension are identical due to multipath reflection, but they can be dealt with by the MDF algorithm. Hence DP-2DHR can be applied as before for joint hop timing and frequency estimation.

Note that frequencies and complex amplitudes of different segments pertaining to a particular path can be associated via their corresponding DOA parameters, since for a single segment, frequency, amplitude, and DOA parameters pertaining to one path are paired up automatically by the MDF algorithm. In addition, different paths pertaining to a particular emitter will result in different DOAs but identical hop frequency sequence and hop timing (recall that time delay is treated as phase shift), hence paths can be associated with emitters by hop sequences, which is a clustering

Table 3: DP-2DHR: DOA estimation in the presence of multipath.

	True DOA	Estimated DOA		
		1st Seg.	2nd Seg.	3rd Seg.
Path 1-1	$6°$	$4.67°$	$7.48°$	$6.33°$
Path 1-2	$14°$	$12.25°$	$14.85°$	$13.55°$
Path 2-1	$25°$	$25.50°$	$24.91°$	$24.17°$

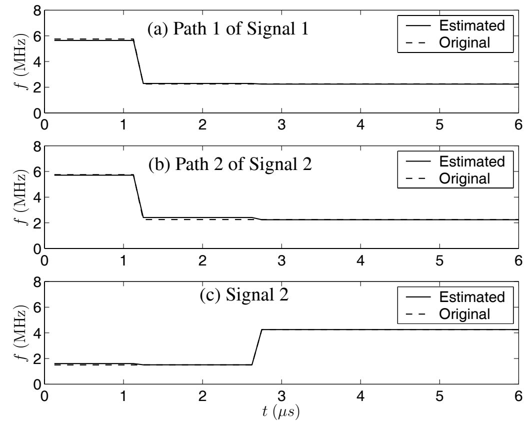

Figure 4: DP-2DHR: multiuser tracking in the presence of multipath.

problem and can be solved, e.g., by calculating the pair-wise distance among all recovered hop sequences.

An example of multiuser tracking by the DP-2DHR algorithm is shown in Table 3 and Figure 4. Again, varying hop frequencies are associated with different paths via their corresponding window invariant DOA parameters. The results show that hop timing and frequency estimates are close to their respective true values. Figure 4 also indicates that paths 1 and 2 pertain to the same emitter since they have essentially the same hop timing and frequency sequence. Similar results have been obtained for GMSK modulated signals [16].

Figure 5 plots the RMSE of hop timing and frequency estimation of the DP-2DHR algorithm in the presence of multipath propagation. The results indicate that the DP-2DHR algorithm performs well in multipath channels, given the fact that the signals are tracked in a situation where hop code, rate, timing, and multipath delay are all unknown.

In practical systems, the number of signals is much less than the number of time samples, and the number of antenna elements usually ranges from 3 to 8. It can be shown that a good estimate of the complexity of the DP-2DHR algorithm is $\mathcal{O}(KN^5)$. There are several ways that this complexity can be reduced: i) It is only during the initial acquisition period that the full complexity of the blind algorithm is needed. If frequencies hop at a regular rate, hop timing and hop period can be estimated by applying the DP-2DHR algorithm to a relatively short portion of a long data record, while frequency estimation for the remaining data can be accomplished by applying the MDF algorithm to pre-decided hop-free data blocks delimited by *system-wide*

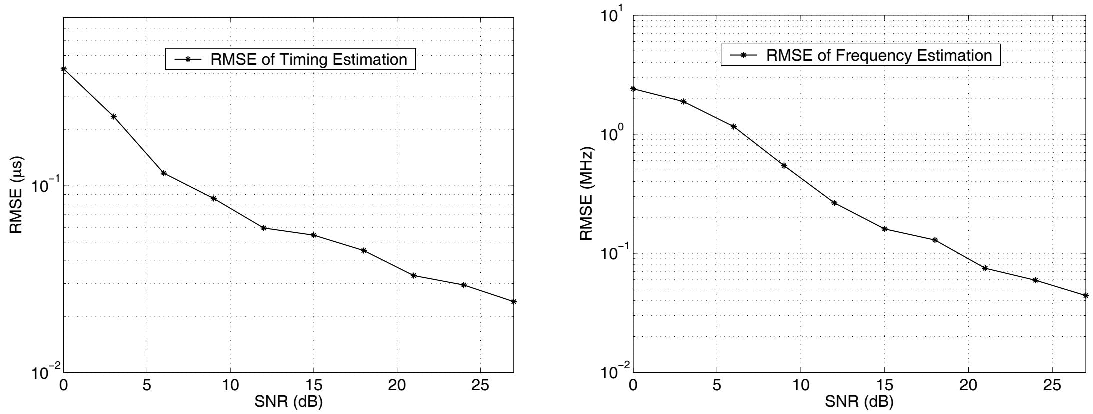

Figure 5: DP-2DHR: RMSE of timing and frequency estimation in the presence of multipath.

adjacent hop instants. This will reduce the complexity significantly. ii) The DP-2DHR algorithm may be simplified using standard approaches of reduced-complexity Viterbi decoding, such as path pruning based on metric thresholding, early path merging, etc. These will of course incur a performance loss, but if, e.g., the truncation parameter is appropriately chosen, the loss will be small. iii) If one has a reasonably good idea about the hop rates, the problem can be much simplified. If the hop code and hop timing of a signal-of-interest are known, then one can de-hop and obtain a model with much reduced noise and interference, since only interferers who collided with the particular user of interest within the observation interval will remain in the de-hopped signal; and the receiver can cut-down its bandwidth to the hopping bin-bandwidth in this case. iv) As a further alternative, simpler frequency estimation techniques can be used in place of 2-D MDF. Clearly, there are many trade-offs one may pursue.

In the development of the DP-2DHR algorithm, signal bandwidth is assumed to be known. In a practical blind estimation scenario, the receiver may also lack knowledge of the signal bandwidth. Relative to the other unknowns (hop patterns, timing and rates), it is simpler for the receiver to estimate the compound signal bandwidth, e.g., via energy detection. However, due to sampling rate and noise power considerations, an intercept receiver may only observe part of the spread bandwidth. In this case, the performance of DP-2DHR will be degraded due to bandwidth mismatch because signals may hop in and out of the observed band, making it difficult to track across hops. Identifiability issues also become much more complicated in the presence of bandwidth mismatch, due to model order variations.

We have conducted extensive simulation experiments to study the performance of the DP-2DHR algorithm. More results may be found in [16]. In addition, based on a software testbed developed for [17], we have designed a new software demo for blind multiuser tracking in FH systems using Matlab. A screen shot of the demo is shown in Figure 6.

B.3 The DP-TALS Algorithm with Multiple Invariance Sensor Arrays

The principle of DP-2DHR can be extended to jointly estimate 2-D DOA (azimuth and elevation angles), hop timing, and frequency of multiple FH transmissions using an antenna array possess-

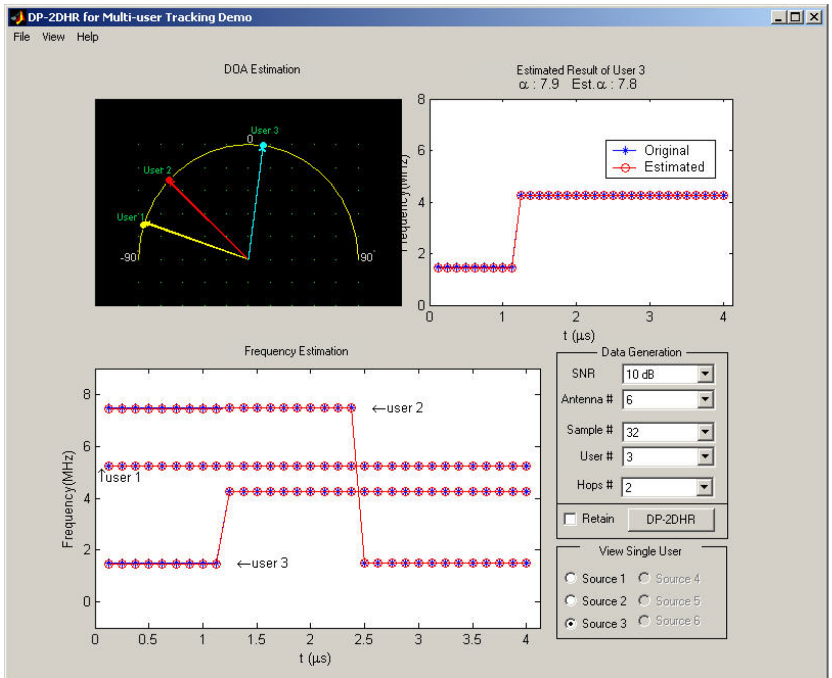

Figure 6: A software demo for blind multiuser tracking in frequency hopping systems.

ing multiple invariances (MI). An MI sensor array is composed of multiple identical subarrays displaced in the same or different directions. A multiple invariant rectangular array is shown in Figure 7. Several methods have been proposed for direction finding and/or other parameter estimation using MI sensor arrays, e.g., [24, 27, 28, 34]. An important assumption of these methods is that the incoming signals are narrowband, so that the propagation delay of the signals from one subarray to another can be approximated by phase shift. However, if the FH system under consideration is a wideband system, then the inherent frequency variability poses special difficulties for signal parameter estimation, due to the fact that the phase shifts among subarrays are (wideband) frequency dependent. Nevertheless, these methods can still be applied to individual hop-free segments if hop timing is known, since for a hop-free data segment, each of the signals impinging on the sensor array can be modeled as a narrowband signal.

Here we extend the idea of DP-2DHR by coupling the DP principle with low-rank decomposition of data collected from an MI sensor array. The resulted algorithm is termed the DP-TALS algorithm where TALS stands for Trilinear Alternating Least Squares. Suppose a receiver utilizes a 2-D antenna array, which is composed of H identical subarrays of m sensors each displaced in different directions. Though each signal's carrier frequencies are hopped over a wide frequency band, between any two *system wide* consecutive hop instants, e.g., n_i and n_{i+1}, the discrete-time baseband equivalent model for the array output can still be written as an $M \times (n_{i+1} - n_i)$ matrix

$$\boldsymbol{X}_i = \mathcal{A}_i \boldsymbol{S}_i^T + \boldsymbol{W}_i, \tag{10}$$

where $\mathcal{A}_i = [\boldsymbol{a}(\alpha_1, \psi_1) \ \cdots \ \boldsymbol{a}(\alpha_d, \psi_d)]$, and α_r, ψ_r are azimuth and elevation angles. The frequency-dependent complex amplitude (due to path attenuation) is absorbed into $\mathcal{A}_i$. Let $\boldsymbol{J}_h$ denote the $m \times M$ selection matrix that extracts the m rows corresponding to the h-th subarray, then it holds

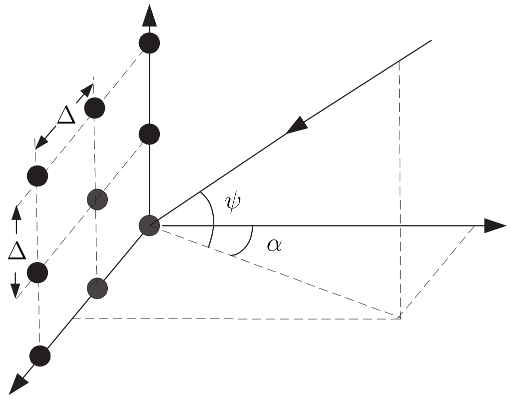

Figure 7: A rectangular sensor array possessing multiple shift invariances.

that [28]

$$\boldsymbol{Y}_i = \begin{bmatrix} \boldsymbol{J}_1 \\ \vdots \\ \boldsymbol{J}_H \end{bmatrix} \boldsymbol{X}_i = \begin{bmatrix} \boldsymbol{A}_i \boldsymbol{\Phi}_i^{(1)} \\ \vdots \\ \boldsymbol{A}_i \boldsymbol{\Phi}_i^{(H)} \end{bmatrix} \boldsymbol{S}_i^T + \boldsymbol{V}_i, \tag{11}$$

where the $m \times d$ matrix $\boldsymbol{A}_i$ is the response of subarray 1 (reference), and $\boldsymbol{\Phi}_i^{(h)}$ is a $d \times d$ diagonal matrix of phase shifts, which is a function of signal parameters (DOA and frequency) and the displacement of the h-th subarray relative to the reference, with $\boldsymbol{\Phi}_i^{(1)} = \boldsymbol{I}$. $\boldsymbol{V}^{(i)}$ is the corresponding noise matrix. Define a $H \times d$ matrix $\boldsymbol{\Phi}_i$ such that its h-th row consists of the diagonal elements of $\boldsymbol{\Phi}_i^{(h)}$, then (11) can be rewritten as

$$\boldsymbol{Y}_i = (\boldsymbol{\Phi}_i \odot \boldsymbol{A}_i)\, \boldsymbol{S}_i^T + \boldsymbol{V}_i, \tag{12}$$

where $\odot$ stands for the Khatri-Rao product.

For given n_i and n_{i+1}, the objective is to blindly estimate 2-D directions α_r and θ_r, as well as frequency $\omega_r^{(p)}$ from $\boldsymbol{Y}_i$, in (12), for $r = 1, \ldots, d$. A key observation is that with proper dimensioning and under certain relatively mild conditions, Eqn. (12) is in fact a low-rank trilinear (*three-way*) model that exhibits strong identifiability properties [23] and can be estimated via well-established iterative least squares algorithms [24]. Low-rank three-way array decomposition is unique under a relatively mild rank-like condition [10]. The identifiability of the model (12) is established in [24].

In particular, TALS can be used to estimate $\boldsymbol{A}_i$, $\boldsymbol{\Phi}_i$, and $\boldsymbol{S}_i$ from the noisy observations $\boldsymbol{Y}_i$. The basic idea of ALS is to update matrices one by one in an alternating fashion during each iteration, conditioned on previously obtained estimates for the remaining matrices [24]. Upon convergence of TALS, and $\boldsymbol{A}_i$, $\boldsymbol{\Phi}_i$, and $\boldsymbol{S}_i$ will be estimated up to scaling and common permutation of columns. The azimuth and elevation angles can then be estimated via simple division from $\boldsymbol{\Phi}_i$, and the temporal frequencies can be estimated from $\boldsymbol{S}_i$ via single 1-D harmonic retrieval techniques (e.g., periodogram) or simple division. Since the permutation of columns is common to all three matrices, $(\alpha_r, \psi_r, \omega_r^{(p)})$ will be paired up automatically by TALS. Notice that both $\boldsymbol{A}_i$ and $\boldsymbol{S}_i$ in (12) are Vandermonde. This constraint can be incorporated into the iteration process of TALS to

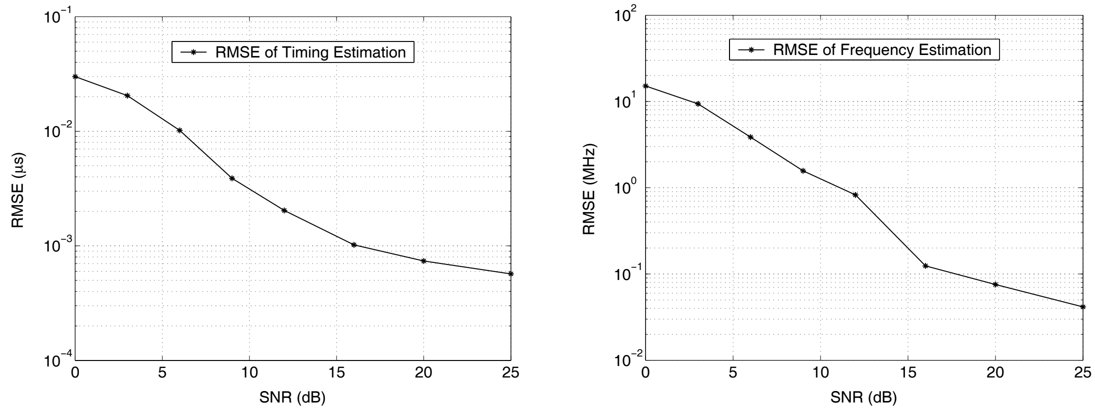

Figure 8: DP-TALS: RMSE of timing and frequency estimation using a 2-D antenna array.

expedite convergence and improve estimation performance. Joint timing and frequency estimation is achieved by coupling DP with TALS (DP-TALS) [13].

We have tested the algorithm with a rectangular array of size 2×6, comprising four overlapping uniform linear subarrays of 5 sensors each. The spacing Δ is half wavelength at $f_c = 2$ GHz. Two BFSK modulated signals with 2-D DOA (elevation α, azimuth ψ) angles of (10°, 6°) and (17°, $12°$), hop with different timing. The hopping bandwidth is 80 MHz with 1 MHz channel spacing. Both symbol period and hop dwell are 0.8 μs. The received signal is sampled at 80 MHz after down-conversion, and $N = 48$ samples are collected at each sensor.

For each realization of the Monte Carlo simulation, each of the two FH-FSK signals hops once within the observation window. Hop timing and frequencies are randomly generated. The RMSE versus SNR curves of the DP-TALS algorithm in Figure 8 demonstrate that the algorithm performs quite well for a wide range of SNRs. However, we note that the complexity of DP-TALS is higher than that of DP-2DHR due to the iterative nature of TALS.

B.4 An EM Algorithm for Hop Timing Estimation

We have shown that the minimizing of (3) is not analytically solvable, and one way to compute it is by dynamic programming, which yield optimal solution, but comes with high computational complexity ($\mathcal{O}(KN^5)$) that may prohibit real-time implementation. The expectation-maximization (EM) principle offers a second alternative to solve this problem. It is more efficient but sometimes only offers sub-optimal solutions since it may converge to a local minimum instead of a globe minimum. The EM algorithm was introduced in the statistics literature as a general approach for iteratively maximizing likelihood functions. It has found applications in many estimation problems in signal processing [19]. For example, the EM algorithm has been proposed for sequence detection and timing synchronization by several researchers [3, 5, 20, 21]. [3] discusses sequence recovery of multiple DS-CDMA users with known synchronization, while [5,20,21] consider joint sequence detection and timing estimation for a single user transmission of baseband or direct sequence CDMA (DS-CDMA) signals. The problem studied here is thus unique in several aspects: we consider joint sequence detection and timing estimation for *multiple FH spread-spectrum* trans-

missions; *frequency collisions* may be present; and the hop pattern is unknown hence matched filtering is not applicable.

To solve (3), let $\boldsymbol{X}$ represent the observed but incomplete data, and $(\boldsymbol{X}, \boldsymbol{n})$ be the complete data. Further define

$$\boldsymbol{\phi} = [\boldsymbol{\theta}^T, \boldsymbol{\beta}^T, \boldsymbol{\omega}^T]^T$$

as the set of parameters to be evaluated. Using the EM principle, the value of $\boldsymbol{\phi}$ which maximizes the likelihood function $p(\boldsymbol{X}|\boldsymbol{\phi})$ corresponds to the value of $\boldsymbol{\phi}$ found iteratively through steps:

E-step: compute

$$Q\left(\boldsymbol{\phi}|\boldsymbol{\phi}^{(p)}\right) = E\left[\log p(\boldsymbol{X}, \boldsymbol{n}|\boldsymbol{\phi})|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right] \tag{13}$$

M-step: compute

$$\boldsymbol{\phi}^{(p+1)} = \arg\max_{\boldsymbol{\phi}} Q\left(\boldsymbol{\phi}|\boldsymbol{\phi}^{(p)}\right) \tag{14}$$

the conditional expectation in the E-step is with respect to the conditional density of the random parameter $\boldsymbol{n}$ given the $\boldsymbol{X}$ and assuming that $\boldsymbol{\phi} = \boldsymbol{\phi}^{(p)}$.

From Bayes rule, it is ready to show that $p(\boldsymbol{X}, \boldsymbol{n}|\boldsymbol{\phi}) = p(\boldsymbol{X}|\boldsymbol{n}, \boldsymbol{\phi})p(\boldsymbol{n}|\boldsymbol{\phi})$. Since the noise is white Gaussian, $p(\boldsymbol{X}|\boldsymbol{n}, \boldsymbol{\phi})$ is a joint Gaussian pdf. Because $\boldsymbol{n}$ is independent to $\boldsymbol{\phi}$, $p(\boldsymbol{n}|\boldsymbol{\phi}) = p(\boldsymbol{n})$. Thus, $E[p(\boldsymbol{n})|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}]$ does not depend on either $\boldsymbol{\phi}$ or $\boldsymbol{\phi}^{(p)}$. After simple manipulation of the log likelihood function and dropping unnecessary terms, we can rewrite the E-step as:

E-step: compute

$$Q\left(\boldsymbol{\phi}|\boldsymbol{\phi}^{(p)}\right) = E\left[\|\boldsymbol{X} - \boldsymbol{D}\|_F^2|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right],$$

where $\boldsymbol{D} = [\boldsymbol{D}_0\ \boldsymbol{D}_1\ \cdots\ \boldsymbol{D}_{K-1}]$, and $\boldsymbol{D}_i = \boldsymbol{A}_i\boldsymbol{B}_i\boldsymbol{S}_i^T$.

To further simplify $Q(\boldsymbol{\phi}|\boldsymbol{\phi}^{(p)})$, let us consider:

$$\|\boldsymbol{X} - \boldsymbol{D}\|_F^2 = \mathrm{tr}\left\{(\boldsymbol{X} - \boldsymbol{D})(\boldsymbol{X} - \boldsymbol{D})^H\right\},$$

therefore

$$\begin{aligned} E\left[\|\boldsymbol{X} - \boldsymbol{D}\|_F^2|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right] = \mathrm{tr}\Big\{&\boldsymbol{X}\boldsymbol{X}^H - E\left[\boldsymbol{D}|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right]\boldsymbol{X}^H \\ &-\boldsymbol{X}E\left[\boldsymbol{D}^H|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right] + E\left[\boldsymbol{D}\boldsymbol{D}^H|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right]\Big\}. \end{aligned} \tag{15}$$

Now let us consider one term of (15). We have

$$\begin{aligned} E\left[\boldsymbol{D}|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right]\boldsymbol{X}^H &= \sum_{i=0}^{K-1} E\left[\boldsymbol{D}_i|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right]\boldsymbol{X}_i^H \\ &= \sum_{i=0}^{K-1} \hat{\boldsymbol{A}}_i^{(p)}\hat{\boldsymbol{B}}_i^{(p)} E\left[\boldsymbol{S}_i^T|\boldsymbol{X}, \boldsymbol{\phi}^{(p)}\right]\boldsymbol{X}_i^H, \end{aligned} \tag{16}$$

since $\hat{\boldsymbol{A}}_i^{(p)}$ and $\hat{\boldsymbol{B}}_i^{(p)}$ do not depend on $\boldsymbol{n}$ if $\boldsymbol{\phi}^{(p)}$ is given. Other expectation terms in (15) can be expanded similarly to (16). Based on (15), as shown in [13], it can be verified that a simplified EM algorithm is given by

E-step: for $1 \le i \le K-1$, compute

$$\hat{n}_i^{(p)} = \arg \min_{n_i \in [\hat{n}_{i-1}^{(p)}+1, \hat{n}_{i+1}^{(p-1)}-1]} \left\| \boldsymbol{X}_i - \hat{\boldsymbol{A}}_i^{(p)} \hat{\boldsymbol{B}}_i^{(p)} (\hat{\boldsymbol{S}}_i^{(p)})^T \right\|_F^2 + \left\| \boldsymbol{X}_{i-1} - \hat{\boldsymbol{A}}_{i-1}^{(p)} \hat{\boldsymbol{B}}_{i-1}^{(p)} (\hat{\boldsymbol{S}}_{i-1}^{(p)})^T \right\|_F^2 \tag{17}$$

M-step: compute

$$\boldsymbol{\phi}^{(p+1)} = \arg\min_{\boldsymbol{\phi}} \left\{ \sum_{i=1}^{K} \left\| \boldsymbol{X}_i - \boldsymbol{A}_i \boldsymbol{B}_i \boldsymbol{S}_i^T \right\|_F^2 |\hat{\boldsymbol{n}}^{(p)} \right\}, \tag{18}$$

where $\boldsymbol{\phi}^{(p+1)}$ can are obtained by applying the MDF algorithm to $\boldsymbol{X}_i$'s determined by $\hat{\boldsymbol{n}}^{(p)}$.

In summary, given a received data block with multiple hops, the EM algorithm first takes a random guess of $\hat{\boldsymbol{n}}^{(0)}$ as the initial for the E-step, and compute $\boldsymbol{\phi}^{(1)}$ as the initial for the M-step. Then the algorithm iterates the following two steps until convergence: the expectation step, Eqn. (17), involves assigning signal segments to the current estimated hop frequencies that fits them best; The maximization step, Eqn. (18), provides a new estimate of hop frequencies, is accomplished by applying the MDF algorithm to data segments according to the updated assignment. Upon convergence, frequencies and complex amplitudes of different segments pertaining to a particular signal can be associated via their corresponding DOA parameters, since for a single segment, frequency and DOA parameters pertaining to one user are paired up automatically by the MDF algorithm. Therefore joint estimation of hop timing and frequencies of all users are obtained.

Figure 9 depicts the RMSE of hop timing and frequency estimation of three FH-FSK signals. For each realization, each of the three FH-FSK signals hops three times within the observation window. Hop timing is randomly generated, and frequencies are also randomly selected from the 80 candidate bins with the constraint that there are always two collisions randomly located in the 10 hop-free segments (in average collision accounts for about 20% time of the observation duration). The RMSE vs. SNR performance shown in these figures indicates that the EM algorithm performs quite well even in the low SNR regime, given the fact that the parameters are estimated in a situation where hop pattern, rate, and timing are all unknown.

B.5 Identifiability of 2-D and Multidimensional Frequency Estimation

The blind multiuser tracking techniques based on the dynamic programming and the EM principles described above require a coupled 2-D frequency estimation step. Therefore the study of identifiability (ID) of 2-D frequency estimation is valuable in determining the maximum number of frequencies (thus the number of FH signals) that can be recovered from a given data size. Recently much progress has been made on the deterministic ID [22] and statistical ID [9,14] of multidimensional frequency estimation. For example, in the 2-D case, the MDF algorithm and the Unitary ESPRIT algorithm can uniquely estimate up to $0.25 M_1 M_2$ frequencies almost surely from a single snapshot of $M_1 \times M_2$ data mixture, provided that the 2-D frequencies are drawn from a continuous distribution (hence the name of statistical identifiability). In this project, we have proved the most relaxed statistical identifiability bound to date, as given in the following theorem [12].

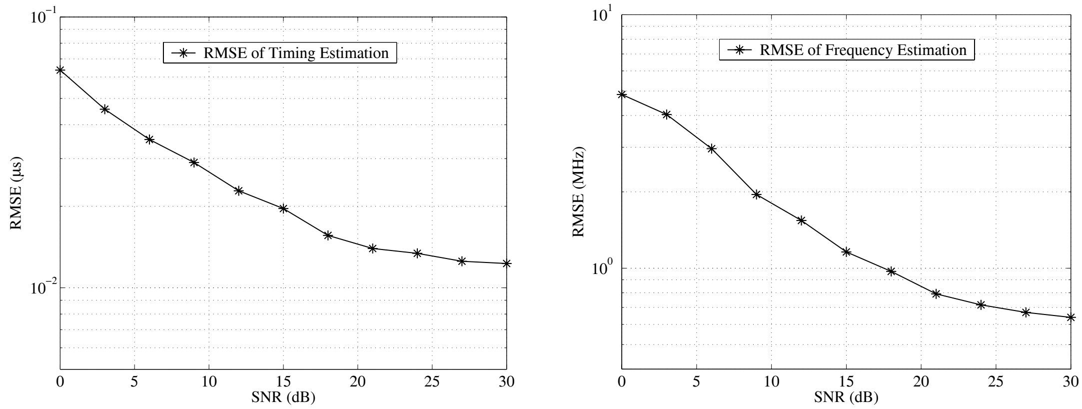

Figure 9: The EM algorithm: RMSE of timing and frequency estimation vs. SNR.

Theorem 1 *Given a sum of F 2-D exponentials as*

$$x_{m_1,m_2} = \sum_{f=1}^{F} c_f e^{j(m_1-1)\omega_f} e^{j(m_2-1)\nu_f}, \tag{19}$$

for $m_1 = 1, \ldots, M_1$, and $m_2 = 1, \ldots, M_2$, and without loss of generality, assume that $M_1 \geq M_2$, then the parameter triples (ω_f, ν_f, c_f), $f = 1, \cdots, F$, are almost surely uniquely resolvable, provided that

$$F \leq \max_{\substack{K_1+L_1=M_1 \\ K_2+L_2=M_2+1}} \min\left(2K_1K_2,\ L_1L_2\right) \tag{20}$$

where $(\omega_f$, $\nu_f)$ are assumed to be drawn from a distribution that is continuous with respect to the Lebesgue measure in Π^{2F}, and c_f is drawn from a continuous distribution on $\mathbb{C}$.

We need the following results to prove Theorem 1. Given (19), define two Vandermonde matrices $\boldsymbol{A} \in \mathbb{C}^{M_1 \times F}$ and $\boldsymbol{B} \in \mathbb{C}^{M_2 \times F}$, with generators $e^{j\omega_f}$ and $e^{j\nu_f}$, $f = 1, \ldots, F$, respectively, then the 2-D mixture in (19) can be written in matrix form as

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{D}(\boldsymbol{c})\boldsymbol{B}^T, \tag{21}$$

where $\boldsymbol{c} = [c_1, c_2, \ldots, c_F]^T$. Eqn. (21) can also be written in vector form. For example, let

$$\boldsymbol{x} = [x_{1,1},\ x_{1,2},\ \cdots,\ x_{1,M_2},\ x_{2,1},\ \cdots,\ x_{M_1,M_2}]^T,$$

then it can be verified that

$$\boldsymbol{x} = (\boldsymbol{A} \odot \boldsymbol{B})\boldsymbol{c}, \tag{22}$$

Proposition 1 *If we define a two-dimensional smoothing operator $\mathcal{S}$ for the measurement vector $\boldsymbol{x}$ in* (22) *as*

$$\mathcal{S}(\boldsymbol{x}) := [\boldsymbol{J}_{1,1}\boldsymbol{x} \cdots \boldsymbol{J}_{1,L_2}\boldsymbol{x}\ \boldsymbol{J}_{2,1}\boldsymbol{x} \cdots \boldsymbol{J}_{2,L_2}\boldsymbol{x} \cdots \boldsymbol{J}_{L_1,L_2}\boldsymbol{x}],$$

where

$$\begin{aligned}
&\boldsymbol{J}_p^{K_1} = \left[\mathbf{0}_{K_1\times(p-1)}\ \boldsymbol{I}_{K_1}\ \mathbf{0}_{K_1\times(L_1-p)}\right] \in \mathbb{C}^{K_1\times M_1},\\
&\boldsymbol{J}_q^{K_2} = \left[\mathbf{0}_{K_2\times(q-1)}\ \boldsymbol{I}_{K_2}\ \mathbf{0}_{K_2\times(L_2-q)}\right] \in \mathbb{C}^{K_2\times M_2},\\
&\boldsymbol{J}_{p,q} = \boldsymbol{J}_p^{K_1} \otimes \boldsymbol{J}_q^{K_2},
\end{aligned} \tag{23}$$

and $1 \le p \le L_1$, $1 \le q \le L_2$, *and* K_i *and* L_i, *for* $i = 1, 2$, *are positive integers such that*

$$K_1 + L_1 = M_1 + 1, \quad K_2 + L_2 = M_2 + 1. \tag{24}$$

Then it can be verified that

$$\boldsymbol{X}_{\mathcal{S}} = \mathcal{S}(\boldsymbol{x}) = \left(\boldsymbol{A}^{(K_1)} \odot \boldsymbol{B}^{(K_2)}\right) \boldsymbol{D}\left(\boldsymbol{c}\right) \left(\boldsymbol{A}^{(L_1)} \odot \boldsymbol{B}^{(L_2)}\right)^T.$$

Next we present the proof of Theorem 1 in the following.

Proof: Given $\boldsymbol{x}$, define two selection matrices:

$$\boldsymbol{J}_1 = \left[\boldsymbol{I}_{M_1-1}\ \mathbf{0}_{(M_1-1)\times 1}\right],\ \boldsymbol{J}_2 = \left[\mathbf{0}_{(M_1-1)\times 1}\ \boldsymbol{I}_{M_1-1}\right].$$

Due to the shift invariance property of Vandermonde matrices, we have

$$\boldsymbol{x}_1 = (\boldsymbol{J}_1 \otimes \boldsymbol{I}_{M_2})\boldsymbol{x} = (\boldsymbol{A}^{(M_1-1)} \odot \boldsymbol{B})\boldsymbol{c}, \tag{25}$$

$$\boldsymbol{x}_2 = (\boldsymbol{J}_2 \otimes \boldsymbol{I}_{M_2})\boldsymbol{x} = (\boldsymbol{A}^{(M_1-1)} \odot \boldsymbol{B})\boldsymbol{D}(\boldsymbol{\omega})\boldsymbol{c}, \tag{26}$$

where $\boldsymbol{x}$ is given in (22), and $\boldsymbol{\omega} := [e^{j\omega_1},\ e^{j\omega_2},\ \cdots,\ e^{j\omega_F}]^T$. We can now apply the 2-D smoothing operator $\mathcal{S}$ defined in Proposition 1 to $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. Since the sizes of $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are $(M_1 - 1) \times M_2$, the integers in (24) should be chosen such that

$$K_1 + L_1 = M_1, \quad K_2 + L_2 = M_2 + 1. \tag{27}$$

Applying the 2-D smoothing operator, we obtain

$$\begin{aligned}
\boldsymbol{X}_{1,\mathcal{S}} &= \mathcal{S}(\boldsymbol{x}_1) = \boldsymbol{G}\boldsymbol{D}(\boldsymbol{c})\boldsymbol{H}^T,\\
\boldsymbol{X}_{2,\mathcal{S}} &= \mathcal{S}(\boldsymbol{x}_2) = \boldsymbol{G}\boldsymbol{D}(\boldsymbol{c})\boldsymbol{D}(\boldsymbol{\omega})\boldsymbol{H}^T,
\end{aligned}$$

where $\boldsymbol{G} := \boldsymbol{A}^{(K_1)} \odot \boldsymbol{B}^{(K_2)}$, and $\boldsymbol{H} := \boldsymbol{A}^{(L_1)} \odot \boldsymbol{B}^{(L_2)}$.

To further explore the data structure, we can perform the backward smoothing on the data vector $\boldsymbol{x}$ in (22). Define

$$\boldsymbol{y} := \boldsymbol{\Pi}\boldsymbol{x}^* = (\boldsymbol{A} \odot \boldsymbol{B})\widetilde{\boldsymbol{c}}, \tag{28}$$

where $\boldsymbol{\Pi}$ is a backward permutation matrix with ones on its antidiagonal, and $\widetilde{\boldsymbol{c}} = [\widetilde{c}_1, \widetilde{c}_2, \cdots, \widetilde{c}_F]^T$, with

$$\widetilde{c}_f = c_f e^{-j(M_1-1)\omega_f - j(M_2-1)\nu_f}. \tag{29}$$

Applying the same technique to $\boldsymbol{y}$ that we used to obtain $\boldsymbol{X}_{1,\mathcal{S}}$ and $\boldsymbol{X}_{2,\mathcal{S}}$ from $\boldsymbol{x}$, we have

$$\begin{gathered}
\boldsymbol{y}_1 = (\boldsymbol{J}_1 \otimes \boldsymbol{I}_{M_2})\boldsymbol{y} = (\boldsymbol{A}^{(M_1-1)} \odot \boldsymbol{B})\widetilde{\boldsymbol{c}}, \\
\boldsymbol{y}_2 = (\boldsymbol{J}_2 \otimes \boldsymbol{I}_{M_2})\boldsymbol{y} = (\boldsymbol{A}^{(M_1-1)} \odot \boldsymbol{B})\boldsymbol{D}(\boldsymbol{\omega})\widetilde{\boldsymbol{c}}, \\
\boldsymbol{Y}_{1,\mathcal{S}} = \mathcal{S}(\boldsymbol{y}_1) = \boldsymbol{G}\boldsymbol{D}(\widetilde{\boldsymbol{c}})\boldsymbol{H}^T, \\
\boldsymbol{Y}_{2,\mathcal{S}} = \mathcal{S}(\boldsymbol{y}_2) = \boldsymbol{G}\boldsymbol{D}(\widetilde{\boldsymbol{c}})\boldsymbol{D}(\boldsymbol{\omega})\boldsymbol{H}^T.
\end{gathered}$$

If we define the following matrices

$$\boldsymbol{Z}_1 := \begin{bmatrix} \boldsymbol{X}_{1,\mathcal{S}} \\ \boldsymbol{Y}_{1,\mathcal{S}} \end{bmatrix}, \quad \boldsymbol{Z}_2 := \begin{bmatrix} \boldsymbol{X}_{2,\mathcal{S}} \\ \boldsymbol{Y}_{2,\mathcal{S}} \end{bmatrix}, \tag{30}$$

then we have

$$\boldsymbol{Z} := \begin{bmatrix} \boldsymbol{Z}_1 \\ \boldsymbol{Z}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{P}\boldsymbol{D}(\boldsymbol{\omega}) \end{bmatrix} \boldsymbol{H}^T, \tag{31}$$

where the size of $\boldsymbol{Z}$ is $4K_1K_2 \times L_1L_2$, and a $2K_1K_2 \times F$ matrix $\boldsymbol{P}$ is defined as $\boldsymbol{P} := [\boldsymbol{c} \ \widetilde{\boldsymbol{c}}]^T \odot \boldsymbol{G}$. Since both $\boldsymbol{P}$ and $\boldsymbol{H}$ are Khatri-Rao products of Vandermonde matrices, invoking the Theorem 1 of [14], when $2K_1K_2 \geq F$ and $L_1L_2 \geq F$, $\boldsymbol{P}$ and $\boldsymbol{H}$ are almost surely full column rank. Hence $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ are of rank F. The singular value decomposition of $\boldsymbol{Z}$ yields

$$\begin{bmatrix} \boldsymbol{Z}_1 \\ \boldsymbol{Z}_2 \end{bmatrix} = \boldsymbol{U}_s \boldsymbol{\Sigma}_s \boldsymbol{V}_s^H, \tag{32}$$

where $\boldsymbol{U}_s$ has F columns which together span the column space of $\boldsymbol{Z}$. Since the same space is spanned by the columns of $[(\boldsymbol{P})^T \ (\boldsymbol{P}\boldsymbol{D}(\boldsymbol{\omega}))^T]^T$, there exist an $F \times F$ nonsingular matrix $\boldsymbol{T}^{-1}$ such that

$$\boldsymbol{U}_s = \begin{bmatrix} \boldsymbol{U}_1 \\ \boldsymbol{U}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{P}\boldsymbol{D}(\boldsymbol{\omega}) \end{bmatrix} \boldsymbol{T}^{-1}. \tag{33}$$

Here we divide $\boldsymbol{U}_s$ into two equal size sub-matrices $\boldsymbol{U}_1$, and $\boldsymbol{U}_2$, each of size $2K_1K_2 \times F$. Then $\boldsymbol{U}_1^\dagger \boldsymbol{U}_2$ satisfies:

$$\boldsymbol{U}_1^\dagger \boldsymbol{U}_2 = \boldsymbol{T}\boldsymbol{D}(\boldsymbol{\omega})\boldsymbol{T}^{-1}. \tag{34}$$

Assuming that the elements of $\boldsymbol{\omega}$ are distinct, $\boldsymbol{T}$ can be obtained from the eigenvalue decomposition of $\boldsymbol{U}_1^\dagger \boldsymbol{U}_2$ up to column permutation and scaling ambiguity. Suppose that the eigenvalue decomposition of $\boldsymbol{U}_1^\dagger \boldsymbol{U}_2$ gives

$$\boldsymbol{T}' = \boldsymbol{T}\boldsymbol{\Lambda}\boldsymbol{\Delta}, \tag{35}$$

where $\boldsymbol{\Lambda}$ is a nonsingular diagonal column scaling matrix and $\boldsymbol{\Delta}$ is a permutation matrix. Because the column scaling and permutation will not have material effect on the algorithm, for notation simplicity we use the same symbol for a matrix with or without scaling and permutation as long as it is clear from the context. Once we obtain $\boldsymbol{T}'$, we can retrieve $\boldsymbol{P}$ and $\boldsymbol{H}$ up to common permutation and column scaling according to

$$\boldsymbol{P}' = \boldsymbol{U}_1 \boldsymbol{T}' = \boldsymbol{P}\boldsymbol{\Lambda}\boldsymbol{\Delta}, \tag{36}$$

$$\boldsymbol{H}' = \left(\boldsymbol{P}'^\dagger \boldsymbol{Z}_1\right)^T = \boldsymbol{H}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Delta}. \tag{37}$$

Table 4: Identifiability bound of 2-D frequency estimation

$M_1 \backslash M_2$	3	4	5	6	7	8	9	10	11	12
3	4	4	6	8	8	10	12	12	14	16
4	4	6	8	9	12	12	15	16	18	20
5	6	8	9	12	12	16	18	20	21	24
6	8	9	12	12	16	18	20	24	24	28
7	8	12	12	16	18	20	24	25	30	32
8	10	12	16	18	20	24	25	30	32	36
9	12	15	18	20	24	25	30	32	36	40
10	12	16	20	24	25	30	32	36	40	42
11	14	18	21	24	30	32	36	40	42	49
12	16	20	24	28	32	36	40	42	49	50

In (37) we have used the fact that $\boldsymbol{\Delta}^{-1} = \boldsymbol{\Delta}^T$. The permutation is not an issue here, however, because ω_f and ν_f appear in the same column of $\boldsymbol{P}'$, as well as in $\boldsymbol{H}'$, thus are automatically paired; and arbitrary nonzero column scaling is immaterial, because the frequencies can be obtained by dividing suitably chosen elements of the aforementioned column. For example, suppose $e^{j\omega_f}$ and $e^{j\nu_f}$ appear in the f-th column of $\boldsymbol{P}'$, then $e^{j\omega_f}$ can be retrieved by any one of the following equations in the noiseless case:

$$e^{j\omega_f} = \frac{p'_{n,f}}{p'_{n-K_2,f}}, \quad n = K_2(k_1 - 1) + k_2, \tag{38}$$

where $k_1 = 2, \cdots, K_1$, $k_2 = 1, \cdots, K_2$, and similarly, $e^{j\nu_f}$ can be retrieved by any one of the following:

$$e^{j\nu_f} = \frac{p'_{n,f}}{p'_{n-1,f}}, \quad n = K_2(k_1 - 1) + k_2, \tag{39}$$

where $k_1 = 1, \cdots, K_1$, and $k_2 = 2, \cdots, K_2$. Notice that $e^{j\omega_f}$ and $e^{j\nu_f}$ are automatically paired since they are obtained from the same column of $\boldsymbol{P}'$.

Hence we have shown that the 2-D frequencies can be uniquely recovered almost surely, provided that $F \leq 2K_1K_2$ and $F \leq L_1L_2$, where K_i and L_i are positive integers subject to

$$K_1 + L_1 = M_1, \quad K_2 + L_2 = M_2 + 1.$$

Therefore Theorem 1 is proved. ■

It is difficult to obtain the exact solution of the integer optimization problem in (20). Here we have listed the maximum number of 2-D frequencies that can be uniquely identified for certain data sizes in Table 4. We have also found the lower and upper bounds of the maximum number identifiable 2-D frequencies as given in the following proposition [12].

Proposition 2 *The maximum number of 2-D frequencies given in Theorem 1 is bounded by*

$$\min\left\{2\left[(\sqrt{2}-1)M_1\right]\left[(\sqrt{2}-1)(M_2+1)\right],\left[(2-\sqrt{2})M_1\right]\left[(2-\sqrt{2})(M_2+1)\right]\right\}$$
$$\leq \max_{\substack{K_1+L_1=M_1\\K_2+L_2=M_2+1}} \min\left(2K_1K_2,\ L_1L_2\right) \ \leq\ \lfloor 0.343M_1(M_2+1)\rfloor\,.$$

Theorem 1 shows significant improvement on the identifiability bound of 2-D frequency estimation over existing algebraic algorithms. Previously the most relaxed statistical ID bound is achieved by the MDF algorithm as shown in [15]. Using Theorem 1 of [14], we are also able to obtain the statistical ID bounds of the Unitary ESPRIT algorithm [6] and the MEMP algorithm [8]. Figure 10 plots a comparison of the statistical ID bounds of different algebraic algorithms for 2-D frequency estimation in the absence of noise. The Unitary ESPRIT algorithm and the MDF algorithm have the same ID bound, which is

$$F \leq \lceil M_1/2\rceil\lceil M_2/2\rceil\,.$$

The statistical ID bound of the MEMP algorithm is slightly smaller than those of the MDF algorithm and the Unitary ESPRIT algorithm because no backward-forward smoothing is used in the MEMP algorithm. Note that the deterministic ID bound of the MEMP algorithm is

$$F \leq \min\left(M_1/2, M_2/2\right)$$

as shown in [8] and [33]. It can be seen from Fig. 10 that Theorem 1 offers a significantly improved ID bound over existing results. We note that an algebraic algorithm for 2-D frequency estimation can be obtained from the constructive proof of Theorem 1, which may be used to replace the aforementioned MDF algorithm [12].

The 2-D identifiability results can be generalized to the case of N-D frequency estimation.

Theorem 2 *Given a sum of F N-D exponentials*

$$x_{m_1,\ldots,m_N} = \sum_{f=1}^{F} c_f \prod_{n=1}^{N} e^{j\omega_{f,n}(m_n-1)}, \tag{40}$$

for $m_n = 1,\ldots,M_n$, $n = 1,\ldots,N$, and without loss of generality, assume that $M_1 = \max\{M_n, n = 1,\ldots,N\}$, if

$$F \leq \max_{\substack{K_1+L_1=M_1\\K_n+L_n=M_n+1\\2\leq n\leq N}} \min\left(2\prod_{n=1}^{N}K_n,\ \prod_{n=1}^{N}L_n\right), \tag{41}$$

and the distributions used to draw the NF frequencies and F amplitudes are continuous with respect to Lebesgue measure in Π^{NF} and $\mathbb{C}$, respectively, then the parameter $(N+1)$-tuples $(\omega_{f,1},\ldots,\omega_{f,N},c_f)$, $f = 1,\ldots,F$, are almost surely uniquely resolvable.

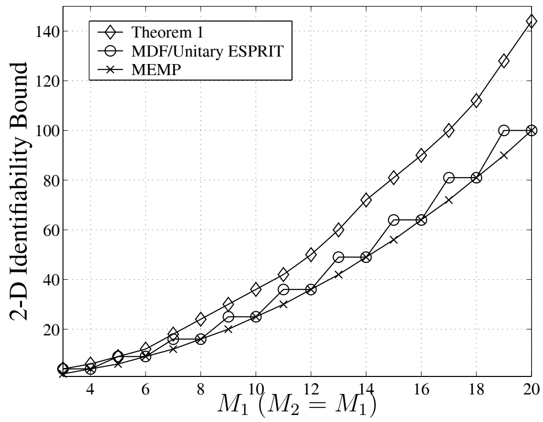

Figure 10: Comparison of identifiability results for 2-D frequency estimation ($M_1 = M_2$).

B.6 Conclusion

In this project we have developed a signal processing scheme for blind tracking of multiple FH signals over multipath channels. This technique is based on the principle of dynamic programming and expectation-maximization, coupled with multidimensional harmonic and low-rank analysis. Numerical simulations demonstrate its capability of joint estimation of hop timing, frequency, and DOA of multiple FH signals in the presence of frequency collisions, without the knowledge of signal hop patterns. The significance of this project in basic research lies in innovative methods for signal detection and jammer localization in frequency hopping communications, and fundamental understanding of the identifiability of multidimensional frequency estimation. Furthermore, it has yielded practical algorithms that are applicable in the presence of modulation uncertainly and unknown channels.

B.7 Bibliography

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C Publications from this Project

(a) Papers published in peer-reviewed journals:

1. X. Liu, N. D. Sidiropoulos, and A. Swami, "Joint hop timing and frequency estimation for collision resolution in frequency hopped networks," *IEEE Trans. Wireless Communications*, vol. 4, no. 6, to appear, Nov. 2005 (submitted Apr. 2004).

(b) Book chapters published:

1. X. Liu, N. D. Sidiropoulos, and T. Jiang, "Multidimensional harmonic retrieval with applications in MIMO wireless channel sounding," in A. B. Gershman and N. D. Sidiropoulos, editors, *Space-Time Processing for MIMO Communications*, Wiley, pp. 41-75, 2005.
2. X. Liu and N. D. Sidiropoulos, "PARAFAC techniques for high-resolution array processing," in Y. Hua, A. Gershman, and Q. Cheng, editors, *High Resolution and Robust Signal Processing*, Marcel Dekker Inc., New York, pp. 111-150, 2003.

(c) Papers published in conference proceedings:

1. J. Liu and X. Liu, "An eigenvector-based algebraic approach for two-dimensional frequency estimation with improved identifiability," *IEEE 6th Workshop on Signal Processing Advances in Wireless Communications*, pp. 655–659, New York City, NY, Jun. 2005.
2. J. Li, X. Liu, and A. Swami, "Detection of model order variation in frequency hopping system with bandwidth mismatch," *IEEE 6th Workshop on Signal Processing Advances in Wireless Communications*, pp. 680–684, New York City, NY, Jun. 2005.
3. X. Ma, S. Zhou and X. Liu, "A novel view on modulus-preserving rate-one space time block codes," *Proc. of the 2005 International Conf. on Wireless Networks, Communications, and Mobile Computing*, Maui, HI, Jun. 2005.
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(d) Manuscripts submitted, but not published:

1. J. Liu and X. Liu, "An eigenvector based algebraic approach for multidimensional frequency estimation with improved identifiability," *IEEE Trans. Signal Processing*, submitted May 2005, revised Oct. 2005.
2. X. Liu, J. Li, and X. Ma, "An EM algorithm for blind hop timing estimation of multiple frequency hopped signals with bandwidth mismatch," *IEEE Trans. Vehicular Technology*, submitted Jun. 2005.

(e) Papers presented at meetings, but not published in conference proceedings:

1. X. Liu, "Code-blind reception of frequency hopped signals over multipath fading channels," at *the International Conference on Acoustics, Speech, and Signal Processing*, Montreal, Quebec, Canada, May 2004.
2. X. Liu, "On constant modulus multidimensional harmonic retrieval," at *the Trilinear Methods in Chemistry and Psychology Conference*, Lexington, KY, Jun. 2003.

(f) Technical reports submitted to ARO:

1. X. Liu, "Signal detection and jammer localization in multipath channels for frequency hopping communications," Technical report submitted to ARO, Oct. 2005.

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2. Jun Liu (graduate student)
3. Xiangqian Liu (principle investigator)

E Inventions and Patents

None.